M5's, D4's and 5D SYM

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(with N. Lambert and M. Schmidt-Sommerfeld, arXiv:1012.2882)

Motivation

Over the last three years there has been significant amount of work towards actions for multiple M2-branes.

Progress relied on the introduction of a novel algebraic structure: a 3-algebra. [Bagger-Lambert, Gustavsson]

These ideas solidified in the ABJM proposal for bifundamental CS-matter theory describing N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ M-theory singularity. [Aharony-Bergman-Jafferis-Maldacena]

Important developments in $\text{AdS}_4/\text{CFT}_3$...

But what about the M5-brane??

Low-energy M5-brane dynamics governed by a theory in 6d with: [Strominger, Witten]

- $(2,0)$ supersymmetry
- conformal invariance
- $\mathrm{SO}(5)$ R-symmetry

The $(2,0)$ tensor multiplet contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions

It is believed to arise from fluctuations of M2-brane ending on the M5 worldvolume: selfdual string soliton.

E.g. take 2 separated M5's with M2's stretched between them. In this broken phase these membranes have tension.

In the limit where the M5's become coincident (symmetric phase) the membranes degenerate into tensionless strings.

At small separations this theory of strings can be decoupled from gravity.

But it's complicated: getting Lagrangian for single M5 difficult because of selfdual three-form field strength.

Note: $\exists$ indirect ways of attacking the abelian problem

- $\diamond$ Sacrificing manifest 6d Lorentz invariance
- $\diamond$ Introducing auxiliary scalar field

[Aganagic-Park-Popescu-Schwarz, Pasti-Sorokin-Tonin, Bandos et al., Belov-Moore]

Can still work at the level of susy xfms and e.o.m..

Relation to 5D SYM

From String Theory point of view relation between D4- and M5-brane theories given by compactification on S^1.

The strong-coupling dynamics of D4-theory should be encoded in M5-theory. There are three distinct theories at play:

- The 6D (2,0) susy M5-brane CFT
- The reduction of this on S^1 to 5D but keeping all KK modes
- The 5D SYM theory that captures the dynamics of D4-branes at weak coupling

A few comments on 5D SYM are in order at this stage:

- ◇ It is naïvely non-renormalisable and as such new d.o.f. should appear at some scale
- ◇ It has a UV fixed-point
- ◇ Nahm's classification of CFT's in various dimensions tells us that the UV-fixed point theory cannot be 5D.
- ◇ It is natural to identify this with the 6D (2,0) CFT. This fits nicely with the String Theory intuition [Seiberg]

5D SYM also has particle states that carry instanton charge k with mass

$$M \propto \frac{k}{g_{YM}^2} \propto \frac{k}{R_5}$$

Simplest such states are just D0's in the D4 worldvolume.

This would match the KK spectrum of compactified 6D theory. [Rozali, Berkooz-Rozali-Seiberg]

$\Rightarrow$ Even in Yang-Mills limit, we can identify a tower of states which seem to know about the extra direction of M-theory.

Conclusions

⇒ All KK modes of (2,0) on S^1 included in 5D SYM. No new d.o.f. need to be added in the UV.

⇒ Its strong coupling limit should give the (2,0) theory

⇒ Implications for renormalisability and finiteness of 5D SYM

An interesting related fact is that 5D SYM is finite up to 5 loops [Bern-Dixon-Dunbar-Grant-Perelstein-Rozowsky]

Make this quantitative by identifying BPS spectrum of 5D SYM and 6D (2,0) theories.

Note: Difficulty in doing so in symmetric phase due to presence of a noncompact zero mode coming from the instanton size modulus $\Rightarrow$ continuous spectrum.

This will be performed in the broken phase, where the instanton size gets stabilised.

$\Rightarrow$ See also complementary work of Douglas arXiv:1012.2880

Outline

- 5D SYM superalgebra
- Particle states with instanton charge
- String / ‘Photon’ / Other states
- Conclusions (again)

5D SYM superalgebra

Begin by writing down the 5D SYM susy transformations

$$\begin{aligned}
\delta_\epsilon X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\delta_\epsilon A_\mu &= i\bar{\epsilon}\Gamma_\mu\Gamma_5\Psi \\
\delta_\epsilon\Psi &= \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\Gamma_5\epsilon + D_\mu X^I\Gamma^\mu\Gamma^I\epsilon - \frac{i}{2}[X^I, X^J]\Gamma^{IJ}\Gamma^5\epsilon
\end{aligned}$$

with $\Gamma_{012345}\epsilon = \epsilon$ and the action

$$\begin{aligned}
S = & -\frac{1}{g_{YM}^2}\int d^5x\,\mathrm{tr}\Big(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu X^I D^\mu X^I - \frac{i}{2}\bar{\Psi}\Gamma^\mu D_\mu\Psi \\
& + \frac{1}{2}\bar{\Psi}\Gamma^5\Gamma^I[X^I, \Psi] - \frac{1}{4}\sum_{I,J}[X^I, X^J]^2\Big)
\end{aligned}$$

Straightforward to calculate the superalgebra

$$\begin{aligned}\{Q_\alpha, Q_\beta\} &= P_\mu(\Gamma^\mu C^{-1})^-_{\alpha\beta} + Z_5(\Gamma^5 C^{-1})^-_{\alpha\beta} + Z^I_\mu(\Gamma^\mu \Gamma^I C^{-1})^-_{\alpha\beta} \\ &\quad + Z^I_5(\Gamma^5\Gamma^I C^{-1})^-_{\alpha\beta} + Z^{IJ}_{\mu\nu\lambda}(\Gamma^{\mu\nu\lambda}\Gamma^{IJ} C^{-1})^-_{\alpha\beta}\end{aligned}$$

Let's list some of the central charges

$$\begin{aligned}Z_5 &= -\frac{1}{8g^2_{YM}}\int d^4x \,\mathrm{tr}(F_{ij}F_{kl}\varepsilon_{ijkl}) \\ Z^I_5 &= \frac{1}{g^2_{YM}}\int d^4x \,\mathrm{tr}(D_iX^IF_{0i} + i[X^I, X^J]D_0X^J) \\ Z^I_i &= \frac{1}{2g^2_{YM}}\int d^4x\partial_j \mathrm{tr}(X^IF_{kl})\varepsilon_{ijkl}\end{aligned}$$

and e.g. for $\langle X^6\rangle \neq 0$ one sees that $Z^6_5 = Q_E$ and $Z^6_i = Q_{Mi}$.

Useful to compare this to (2,0) superalgebra in 6D [Howe-Lambert-West]

$$\{Q_\alpha, Q_\beta\} = P_m(\Gamma^m C^{-1})^-_{\alpha\beta} + Z^I_m(\Gamma^m \Gamma^I C^{-1})^-_{\alpha\beta} + Z^{IJ}_{mnp}(\Gamma^{mnp}\Gamma^{IJ}C^{-1})^-_{\alpha\beta}$$

and in particular with its dimensional reduction on S^1.

$\Rightarrow$ Identify $P_5 = Z_5$ and hence:

$$P_5 = \frac{k}{R_5} = -\frac{1}{8g^2_{YM}} \int d^4x \, \mathrm{tr}(F_{ij}F_{kl}\varepsilon_{ijkl})$$

Since both KK momentum and instanton number are quantised:

$$R_5 = \frac{g^2_{YM}}{4\pi^2}$$

Particle states with Instanton charge

Next look for explicit time-independent solutions with $\langle X^6 \rangle \neq 0$. The energy is:

$$\begin{aligned} E &= \frac{1}{g_{YM}^2}\int d^4x \,\mathrm{tr}\Big[\frac{1}{4}F_{ij}F_{ij} + \frac{1}{2}F_{0i}F_{0i} + \frac{1}{2}D_iX^6D_iX^6\Big] \\ &= \frac{1}{g_{YM}^2}\int d^4x \,\mathrm{tr}\Big[\frac{1}{8}(F_{ij} - \frac{1}{2}\varepsilon_{ijkl}F_{kl})^2 + \frac{1}{2}(F_{0i} + D_iX^6)^2 \\ &\qquad + \frac{1}{8}\varepsilon_{ijkl}F_{ij}F_{kl} - D_iX^6F_{0i}\Big] \end{aligned}$$

It is easy to see that this is minimised by

$$F_{ij} = \frac{1}{2}\varepsilon_{ijkl}F_{kl}\ , \qquad D_iX^6 = -F_{0i}\ , \qquad D_iD_iX^6 = 0$$

For $\mathrm{SU}(2)$ these can be solved by

$$A_i = 2\frac{\rho^2}{x^2(x^2+\rho^2)}\eta^a_{ij}x^j\frac{\sigma^a}{2}\ , \qquad X^6 = v\frac{x^2}{x^2+\rho^2}\frac{\sigma^3}{2}$$

Then the electric charge reads

$$Q_E = \frac{1}{g^2_{YM}}\oint \mathrm{tr}(\langle X^6\rangle F_{0i})d^3S_i = v^2\rho^2 g^{-2}_{YM}$$

Thus the instanton size ρ must in fact be quantised and is no longer a modulus.

The mass is given by

$$M = |Z_5| + |Q_E|$$

These solutions preserve 1/4 of the susy since one has

$$\delta\Psi = F_{0i}\Gamma_i(\Gamma_0\Gamma_5 - \Gamma_6)\epsilon + \frac{1}{4}F_{ij}\Gamma_{ij}\Gamma_5(1 - \Gamma_{1234})\epsilon$$

which is zero if

$$\Gamma_{056}\epsilon = \epsilon\ , \qquad \Gamma_{05}\epsilon = \epsilon$$

These are the ‘dyonic instanton’ solutions. [Lambert-Tong]

Compare against predictions of $(2,0)$ theory for BPS states with mass M and momentum P_5, i.e. self-dual strings wrapping x^5.

Upon reduction to 5D the electric-central charge of the Yang-Mills theory comes from Z_5^6.

The other central charges are not carried by particle states.

In this case we find

$$\{Q_\alpha, Q_\beta\} = (M + P_5\Gamma^{50} + Z_5^6\Gamma^5\Gamma^6\Gamma^0)^-_{\alpha\beta}$$

Need this to act on simultaneous eigenstates of Γ_{05} and $\Gamma_{05}\Gamma^6$ and since $[\Gamma_{05}, \Gamma_{05}\Gamma^6] = 0$ only 1/4 susy survives.

Recover same BPS formula for the mass.

We can also explore the multiplet structure for the 1/4-BPS 'dyonic instanton' solutions.

There are 12 fermionic zero modes that will lead to a degenerate multiplet with 64 states.

The 12 broken susies fall under representations of:

- The little group for massive states in 5D: $Spin(4) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2)$
- The R-symmetry group: $Spin(5) \to Spin(4) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2)$

The resulting field content for the massive multiplet is:

Fermions: a chiral ψ_{ij}^{+} $(\mathbf{1},\mathbf{1})$, 5 chiral λ $((\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{2}))$ and 8 anti-chiral χ $((\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{2}) \oplus (\mathbf{3},\mathbf{1}))$.

Bosons: a complex doublet of self-dual 2-forms B_{ij}^{+} $(\mathbf{2},\mathbf{1})$, four complex vectors A_i $(\mathbf{1},\mathbf{2}) \oplus (\mathbf{2},\mathbf{1})$ and 10 scalars ϕ $((\mathbf{3},\mathbf{2}) \oplus (\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2}))$.

Note: Highest weight state is a fermion. Bosons transform under spinor representations of $\mathrm{SO}(4)$ R-symmetry making spacetime interpretation obscure.

String states with Instanton charge

Also need to account for self-dual strings that do not wrap M-theory direction x^5.

The D4-brane theory has 1/2-BPS string states, corresponding to D2-branes ending on the D4.

From M5-brane picture these states should also allow for generalisations that carry instanton charge.

We find 1/2-BPS solutions with magnetic charge Q_{M4} and instanton charge k

Moduli space only has 3 non-compact modes corresponding to translations transverse to the self-dual string in 6D.

$\Rightarrow$ No issue with obtaining continuous spectrum of states.

Compare again with the 6D $(2,0)$ predictions for state with central charge Z_4^6 and momentum P_5.

In total find a 1/2-BPS state in agreement with the 5D analysis.

‘Photon’ states

Identified massive instanton states corresponding to self-dual strings with E + M charges (‘W-boson’ states).

But 5D SYM also has massless uncharged photon states. KK tower for these modes? There are no smooth instanton solutions in the broken phase.

Problem: $\exists$ massless uncharged photon modes in 5D SYM that propagate in x^i, $i = 1, 2, 3, 4$ but apparently not in x^5. This seems to break 6D Lorentz invariance.

Way out? Little is known about (2,0) theories and possible that the Higgs mechanism is different.

In fact at the level of abelian theory we can see that the KK tower of photon modes can be gauged away.

$\Rightarrow$ Consistent with lack of candidate for tower of uncharged instanton states.

$\Rightarrow$ Suggests that instanton spectrum of 5D SYM is in exact agreement with the spectrum of states of $(2,0)$ theory.

Other states

Also try to look for KK states that carry Z^{IJ}_{ijk} charges. These come from intersections of M5 with another M5.

$\Rightarrow$ 3-brane soliton on worldvolume of original M5.
[Howe-Lambert-West]

But note: an M2-brane can end on the M5 and deposit charge but M5-branes cannot end on other M5's. Not clear if M5-M5 intersections lead to fundamental state in the theory.

We do not find such solutions that have a non-zero instanton number.

Conclusions (again)

- We reviewed the relation between the 6D (2,0) theory and 5D SYM
- Proposed that the (2,0) theory on S^1 is exactly 5D SYM for any value of the coupling
- Gave evidence by matching KK spectrum for charged BPS self-dual string states to instanton solitons of 5D SYM in broken phase

- Also noted that there should be a novel Higgs mechanism for non-Abelian 2-form implying that there is no KK tower of uncharged states.
- This is consistent with their absence from the instanton sector of five-dimensional super-Yang-Mills.
- M5-brane CFT is finite. Should remain finite once compactified.
- Relationship between $(2,0)$ and SYM finiteness may not be so simple: 6D CFT contains momentum states which are non-perturbative from the point of view of the 5D theory.

Outstanding

- Generalise the solutions beyond $\mathrm{SU}(2)$
- Investigate the conformal phase
- Explain A-D-E
- N^3 scaling
- Use these results to attack M5-brane